# A Model for the Time Response of Solid-embedded Thermocouples

by Y. Rabin and D. Rittel

ABSTRACT—Unlike the transient response of a fluid-immersed thermocouple, and in contrast to common belief, the time response of a solid-embedded thermocouple is far from being similar to that of a first-order process. The current study arises from efforts to characterize the transient response of a solid-embedded thermocouple as a result of a steplike temperature change of the measured domain. Results of this study suggest that the response function of the thermocouple is nearly exponentially dependent on the square root of Fourier number (dimensionless time). It follows that, with respect to fluid temperature measurements, significantly faster time response is expected at the initiation of the process on one hand, and much longer time is required for reaching a steady-state temperature on the other hand. It is shown that the thermal diffusivity of the thermocouple is required to be at least one order of magnitude higher than that of the measured domain in order to obtain meaningful results in transient measurements.

KEY WORDS—Thermocouple, time response, mathematical analysis, solid domain

## Introduction

The applications of solid-embedded thermocouples for temperature measurements are numerous. The transient response of the thermocouple is a crucial parameter for thermal design and analyses of rapid-response processes such as the thermal effects associated with stress waves.[1]

The effect of thermal inertia plays an important role in the uncertainty analysis of temperature measurements using thermocouples. The thermal inertia effect causes some undesired delay in the thermocouple response to a temperature change of its surroundings. The magnitude of this delay depends on the rate of change of the surrounding temperature, the geometry of the thermocouple and the thermophysical properties of both the thermocouple and the measured substance. The delay in thermocouple response is a key parameter for the analysis of uncertainty and, thus, for experimental design.

The time response of thermocouples in fluid temperature measurements has been extensively studied and is widely reported in the literature.[2–4] The current study focuses on the complementary analysis; that is, the time response of embedded thermocouples in a solid domain. More specifically, the current study arises from efforts to characterize the transient response of a solid-embedded thermocouple as a result of a sudden temperature change of the measured domain, and to compare it with the well studied, albeit somewhat similar, response of thermocouples in a fluid domain. While this study deals with one source of uncertainty in temperature measurements (i.e., the deviation of the thermocouple's temperature from its surrounding temperature due to the effect of thermal inertia), there are other sources that contribute to the uncertainty interval, such as electrical amplifiers, analog-to-digital converters, surrounding temperature compensation, the presence of electrical/magnetic fields, random ground currents and so on.[5–7]

## Mathematical Analysis

Fortunately, the heat transfer problem of a fluid-immersed thermocouple has a closed-form solution for idealized conditions. The availability of this solution has led to the concept of the thermocouple time constant, which characterizes the thermocouple's time response. The value of the thermocouple time constant has become the single most important parameter in thermocouple selection for experimental design of rapid-response thermal systems. The well-studied response of a fluid-immersed thermocouple is described in detail here, followed by our new analysis for a solid-embedded thermocouple.

### *Analysis of a Fluid-immersed Thermocouple*

The closed-form solution of a fluid-immersed thermocouple corresponds to a sudden immersion of the thermocouple's junction and to a low thermal resistance to heat transfer by conduction within the thermocouple's cross section. The sudden immersion of the thermocouple is modeled as a steplike temperature change of the surrounding temperature. The low thermal resistance to heat transfer by conduction within the thermocouple cross section is measured with respect to the thermal resistance to heat transfer by convection of the surrounding fluid. Biot number $Bi$ is a dimensionless number indicating the above thermal resistance ratio:

$$Bi \equiv \frac{hV}{kA}, \tag{1}$$

where $h$ is the heat transfer coefficient by convection, $V$ is the volume of the immersed portion of the thermocouple, $k$ is the thermal conductivity of the thermocouple and $A$ is the surface area of the thermocouple in contact with the fluid. In a case where the Biot number is less than 0.1, one may assume a low thermal resistance to heat transfer by conduction,

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which results in a uniform temperature distribution within the thermocouple cross section. In this case, the thermocouple can be modeled as what is known as a lumped heat capacity system.[8] Solving the energy balance equation for such a system yields the transient response of the thermocouple:

$$\frac{Tb - T_\infty}{T_0 - T_\infty} = \exp\left(-\frac{hA}{CV}t\right), \tag{2}$$

where $T_b$ is the bulk (volume average) temperature of the thermocouple, $T_0$ is the initial temperature of the thermocouple, $T_\infty$ is the surrounding temperature (fluid temperature), $C$ is the volumetric specific heat of the thermocouple and $t$ is the time.

The exponentially decaying solution presented in eq (2) is similar to a transient response of a first-order process whose time constant $\tau$ is defined by

$$\tau = \frac{CV}{hA}. \tag{3}$$

Accordingly, the thermocouple bulk temperature deviates by 36.8 percent from the new surrounding temperature after a period equal to one time constant (the value of $e^{-1}$). It follows that this deviation is 13.5 percent and 5 percent after two and three time constants, respectively (the value of $e^{-2}$ and $e^{-3}$, respectively).

The concept of a time constant defined by a 63.2-percent response to a steplike temperature change of the surroundings has become widely accepted by experimentalists (a 63.2-percent response corresponds to a 36.8-percent deviation from a steady-state condition). This time constant has become a standard measure for the transient response of thermocouples, and is often evaluated experimentally.[4] The value of the above time constant plays an important role in the uncertainty analysis where, in the general case, the time constant of the thermocouple has to be significantly smaller than the typical time scale of events of the measured process. However, in the special case of a preknown transient response of the thermocouple, meaningful temperature readings can be obtained within shorter periods than the typical time constant of the thermocouple.[1]

### *Analysis of a Solid-embedded Thermocouple*

Unfortunately, the heat transfer solution of a solid-embedded thermocouple has no closed-form solution on one hand and has different characteristics from the solution presented in eq (2) on the other hand. The heat transfer in this case is assumed to be governed solely by conduction and is presented in a dimensionless form:

$$\frac{1}{Fo_i}\frac{\partial T_i^*}{\partial t^*} = \nabla^2 T_i^* \quad i = \begin{cases} TC & \text{Thermocouple} \\ D & \text{Measured Domain} \end{cases}, \tag{4}$$

where $t^*$ is a dimensionless time, $T^*$ is a dimensionless temperature and $Fo$ is the Fourier number (also known as the dimensionless time):

$$Fo_i = \frac{a_i t}{R^2} \quad i = \begin{cases} TC & \text{Thermocouple} \\ D & \text{Measured Domain} \end{cases}, \tag{5}$$

where $\alpha$ is the thermal diffusivity and $R$ is a characteristic length. The characteristic length corresponds to the direction of heat flow, which is either the radius of the thermocouple junction (modeled as a sphere) or the radius of the thermocouple wire (modeled as an infinite cylinder).

Heat conduction with no thermal resistance to heat flow is assumed at the thermocouple surface:

$$-k_D \left.\frac{\partial T_D}{\partial r}\right|_{r=R} = -k_{TC} \left.\frac{\partial T_{TC}}{\partial r}\right|_{r=R}, \tag{6}$$

where $r$ is the radius.

For the purpose of the current analysis, an initial uniform temperature distribution is assumed within both the measured domain and the thermocouple. A steplike temperature change from the temperature $T_0$ to the temperature $T_1$, throughout the measured domain only, is assumed at the initiation of the process

$$T(r, t = 0) = \begin{cases} T_0 & r < R \\ T_1 & r > R \end{cases}, \tag{7}$$

and heat transfer by conduction is assumed to prevail freely thereafter.

The steplike change of the domain temperature [eq (7)] is an idealization of a very rapid heat generation process of high intensity, which takes place for a short period of time. The term *short period of time* is used here with respect to the expected time response of the temperature sensor. Examples for such processes can be found in the literature dealing with fracture formations,[9] stress waves,[1] freezing of biological tissue during cryosurgery[10] and so on. Furthermore, a steplike temperature change of the measured domain leads to the maximal time delay in thermocouple response and is therefore analyzed.

The time response of the solid-embedded thermocouple can be found implicitly by solving eqs (4)-(7) numerically. Nevertheless, an analytical approximation of the above time response is of great value. This approximation can be applied for measuring ultrarapid response processes for cases in which the typical time constant of the process is of the same order of magnitude as the thermocouple or even much smaller. Then, the measured signal can be deconvolved with respect to the impulse response of the sensor in order to assess the original signal. Such a procedure has been applied for infrared sensing[9] and for embedded thermocouples.[1] The closed-form solution of the fluid-immersed thermocouple [eq (2)] has motivated us to seek an approximation for the time response of a solid-embedded thermocouple of the form

$$\frac{T_b - T_1}{T_0 - T_1} = \exp\left[-B \times \left(\frac{\alpha_D t}{R^2}\right)^n\right], \tag{8}$$

where $T_b$ is the bulk (volume average) temperature, $T_0$ and $T_1$ are defined in eq (7) and $B$ and $n$ are empirical coefficients (found from computer experiments). Obviously, eq (8) represents a first-order process for $n = 1$.

## Results and Discussion

The heat transfer problem by conduction of a solid-embedded thermocouple is characterized by two dimensionless parameters only, which are the Fourier numbers of both the thermocouple and the measured domain [eq (4)]. Therefore, the results can be well presented (in the mathematical sense) as a function of ratio of these numbers and one of these parameters independently. Since both regions have the same characteristic length and time, the ratio of the Fourier numbers is simply the ratio of the thermal diffusivities: $\alpha_{TC}/\alpha_D$.

### The Order of Response

The heat transfer problem was solved using a finite differences numerical scheme, which is described in detail in Ref. 11. The coefficients $B$ and $n$ of eq (8) were calculated based on the numerical solution results and using a least squares approximation technique. The best-fit coefficients of eq (8), listed in Table 1, lead to a maximal error in temperature estimation of no larger than 4 percent. The best fit was evaluated up to 63.2 percent of the thermocouple response. Some representative time response curves are shown in Fig. 1.

From Table 1 and Fig. 1, it can be seen that the time response of a solid-embedded thermocouple is by no means similar to that of a first-order process. This transient response is approximately exponentially dependent on the square root of the dimensionless time $Fo$. It means that the short-term response in such a process is much faster in relation to a first-order process. This can easily be observed at the initiation, when the rate of temperature change [i.e., the time derivative of eq (8)] has an infinite value for $n$ of less than one. On the other hand, the time required for reaching a steady-state temperature is relatively longer than that in a first-order process.

It can further be seen from Table 1 that the coefficients $B$ and $n$ approach a constant value with the increase of the thermal diffusivities ratio; say, greater than the order of 10. This means that the transient response of the thermocouple is solely governed by $Fo_D$ in this case; for example, by the radius of the thermocouple and the thermal diffusivity of the measured domain [eq (8)]. This observation is at odds with the commonly accepted assumption that the transient response of a thermocouple is governed by its thermal diffusivity and its radius, regardless of the thermal diffusivity of the measured domain.[1,12]

### The Time Constant

In analogy to a first-order process, we define the time constant $\tau_0$ as the time at which the exponent value of eq (8) equals one:

$$\tau_0 = \frac{R^{2n}}{B\alpha_D^n}. \tag{9}$$

For the purpose of the current discussion, we define two additional time constants that represent the instants at which the center of the thermocouple and the outer surface of the thermocouple experience 63.2-percent response, $\tau_1$ and $\tau_2$, respectively.

Figure 2 shows the dimensionless time $Fo_D$ for the above time constants. It can be seen from Fig. 2 that the difference between the three time constants described above is relatively small for a thermal diffusivity ratio of 10 and vanishes with the increase of the thermal diffusivities ratio. It means that for a high thermal diffusivities ratio, a uniform temperature distribution can be assumed within any cross section of the thermocouple after a period equal to one time constant, whether it is $\tau_0$, $\tau_1$ or $\tau_2$.

It can be seen from Fig. 2 that as the thermal diffusivities ratio increases, $Fo_D$ approaches a constant value that equals 0.0937 in the spherical case and 0.244 in the cylindrical case. As has been pointed out earlier in the discussion, it can be concluded that the time constant is solely dependent on the thermal diffusivity of the measured domain for a high thermal diffusivities ratio.

### The Cross-sectional Temperature Distribution

It is typically assumed that the thermocouple junction, or, alternatively, the cross section of the thermocouple wires, possesses a uniform temperature distribution (in analogy with a low Biot number of the fluid-immersed thermocouple case). This assumption is convenient for the analysis of the Seebeck effect within the thermocouple, which is the effect of the electrical potential gradient generated by a temperature gradient in an open electrical circuit. In the case of a uniform temperature distribution within the cross section of the

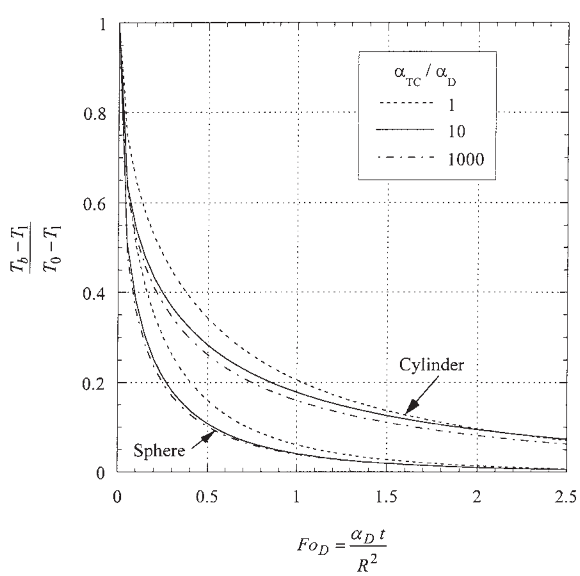


Fig. 1—The transient bulk temperature of the thermocouple junction (modeled as a sphere) and the thermocouple wire (modeled as an infinite cylinder)

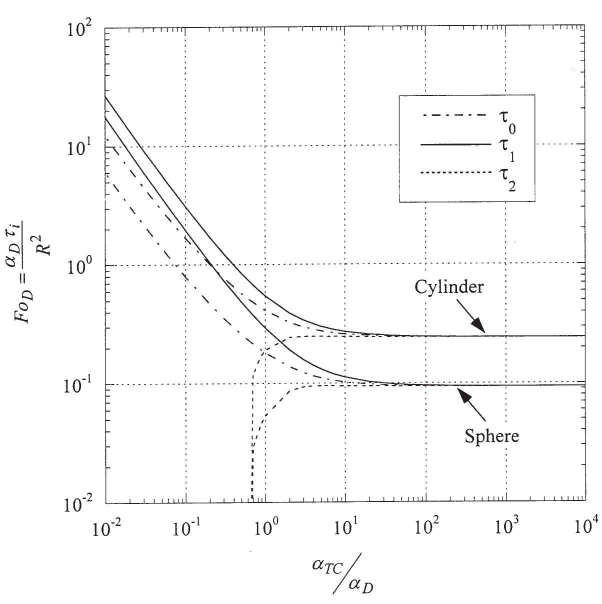


Fig. 2—Fourier number at which a 63.2-percent response to a steplike temperature change is calculated, where $\tau_0$, $\tau_1$ and $\tau_2$ correspond to the bulk temperature, the temperature at the center of the thermocouple and the outer surface temperature, respectively

TABLE 1—BEST-FIT COEFFICIENTS $B$ AND $n$ OF THE RESPONSE FUNCTION [eq (8)]

| | Cylindrical Case | | Spherical Case | |
|---|---|---|---|---|
| $a_{TC}/a_D$ | $B$ | $n$ | $B$ | $n$ |
| 1 | 1.582 | 0.56 | 2.799 | 0.61 |
| 10 | 1.724 | 0.45 | 3.193 | 0.52 |
| 100 | 1.821 | 0.45 | 3.209 | 0.5 |
| 300 | 1.830 | 0.45 | 3.229 | 0.5 |
| 1000 | 1.833 | 0.45 | 3.236 | 0.5 |

thermocouple wires, the electrical potential between the thermocouple wires is nearly linearly dependent on the temperature difference between the thermocouple junction and the point at which the electrical potential is measured. However, in the case of a nonuniform temperature distribution within the cross section of the thermocouple wires, the correlation between the measured electrical potential and the above temperature difference is not well defined. Contrary to previous analyses,[2–4,6,7] the current study does not rely on the assumption of a uniform temperature distribution within the cross section of the thermocouple wires. Instead, the radial temperature distribution within the thermocouple is numerically solved.

It has been concluded from Fig. 2 that the temperature distribution within the thermocouple cross section can be assumed uniform for a high thermal diffusivities ratio and after one time constant. To gain a better understanding of the process, Figs. 3 and 4 show the radial temperature distribution at various points in time for the spherical case (which yields the minimal time constants with respect to the cylindrical case). Figure 3 shows the temperature distribution every $Fo_{TC} = 0.1$ for a thermal diffusivity ratio of one. It can be seen from Fig. 3 that the temperature distribution within the thermocouple cross section cannot be treated as uniform, and thus the correlation of the temperature field with measured electric potential is not clear in this case. The electrical potential readings in this case are not straightforwardly related to the thermocouple average temperature. It can be concluded that a thermal diffusivities ratio of the order of one or less is not appropriate for transient response measurements.

By contrast, an almost uniform temperature distribution within the thermocouple cross section can be observed in Fig. 4, along most of the transient response, for the case of a thermal diffusivity ratio of 300 (e.g., measuring a polymer specimen using a copper-constantan thermocouple). Note that the significant temperature gradients are all in the measured domain in this case. This observation also supports the conclusion previously determined: the transient response is dominated by the measured domain for a high thermal diffusivities ratio.

The volume affected by the presence of the thermocouple is addressed next. Figures 3 and 4 show that there are no significant temperature changes at a distance greater than three times the thermocouple radius for the spherical geometry. Thus, the measured region can be treated as an infinite domain from heat transfer considerations once it has a radius greater than three times the thermocouple radius (a radius 3 times as large bounds a volume 27 times as large).

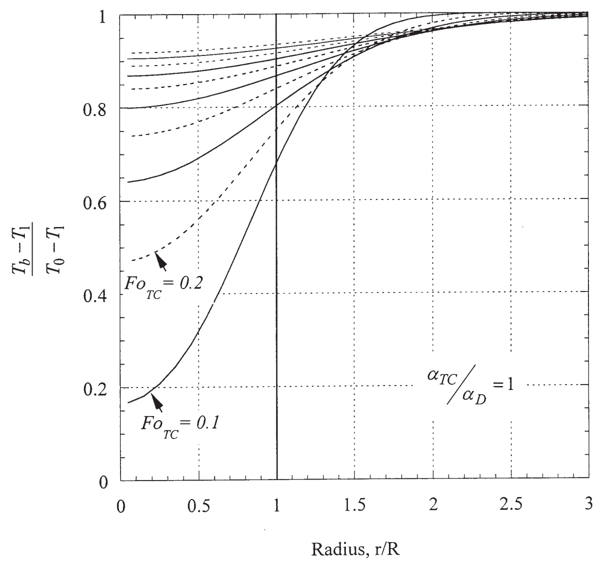


Fig. 3—Radial temperature distribution for the case of $\alpha_{TC}/\alpha_D = 1$; representative curves are shown in $Fo_{TC} = 0.1$ increments

## *Numerical Examples*

### EXAMPLE 1

Assume an experimental setup for measuring the temperature elevation associated with induction heating of a large chunk of 1.2-percent carbon steel using a copper-constantan thermocouple. Induction heating is a volumetric heating process that is derived by the presence of a magnetic material within a transient magnetic field. The thermal diffusivities of the copper wire of the thermocouple and of the carbon steel are about $1.1 \times 10^{-4}$ m$^2$/s and $1.15 \times 10^{-5}$ m$^2$/s, respectively, which yields a thermal diffusivities ratio of about 10. Figure 2 shows relatively uniform temperature distribution within the thermocouple cross section after one time constant, where the time constants for a sphere and an infinite cylinder comply with $Fo_D = 0.1$ and $Fo_D = 0.25$ (based on the bulk temperature), respectively. Using a thin wire thermocouple having a diameter of 0.5 mm, the time constant is found to be $\tau_0 = 5.4 \times 10^{-4}$ s and $\tau_0 = 1.4 \times 10^{-3}$ s, or 0.54 and 1.4 ms, for the sphere and the infinite cylinder cases, respectively.

For comparison purposes, the same thermocouple is now assumed to measure the temperature of air in natural convection and of boiling water in a container ($C = 3.43 \times 10^6$ MJ/m$^3$-°C for copper). Representative values of 15 and 15,000 W/m$^2$-°C are assumed for the heat transfer coefficient by convection in these cases, respectively. Assuming a spherical geometry, the time constant is found to be 19 s for the natural convection case and 19 ms for the boiling water case. Assuming a cylindrical geometry, the same constants are found to be 29 s and 29 ms, respectively. It can be seen that the time constant of a copper-constantan thermocouple in boiling water is at least one order of magnitude larger than that of a steel embedded thermocouple.

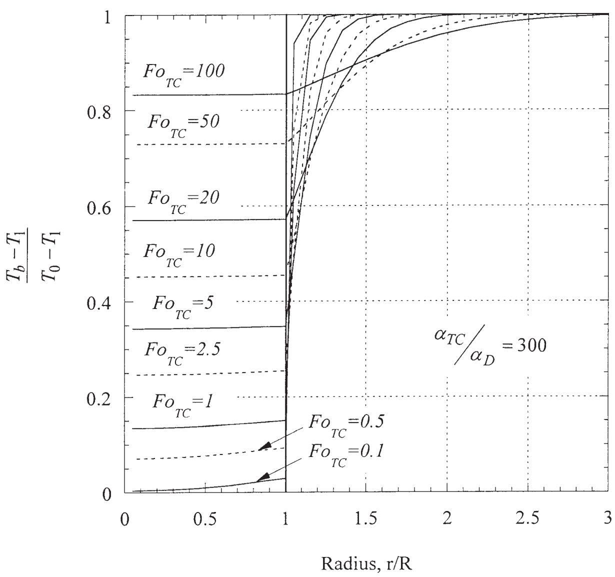


Fig. 4—Radial temperature distribution for the case of $\alpha_{TC}/\alpha_D = 300$ (typical when measuring a polymer specimen using a copper-constantan thermocouple)

### Example 2

Assume an experimental setup for measuring the heating effect of a stress wave within a polymer specimen using a copper-constantan thermocouple.[1] The thermal diffusivities of the copper wire of the thermocouple and of the polymer are about $1.1 \times 10^{-4}$ m$^2$/s and $3.6 \times 10^{-7}$ m$^2$/s, respectively, which yields a thermal diffusivities ratio of about 300. Under these conditions, the time constant of an infinite cylinder complies with $Fo_D = 0.0937$ (Fig. 2). Using an extremely thin wire thermocouple having a diameter of 0.1 mm, one finds the time constant to be $\tau = 6.5 \times 10^{-4}$ s, or 0.65 ms.

One of the basic assumptions for the current analysis is that the duration of heating at the initiation of the process is much shorter than the typical time duration of the thermocouple response. The duration of heating associated with stress waves is of the order of 0.01 to 0.1 ms, which is much shorter than that of the time constant of the thermocouple. Therefore, the heating period in this example can be modeled as a steplike temperature change of the measured domain with respect to the time response of the thermocouple. Nevertheless, and as previously mentioned, when actual signals are analyzed, the situation is not that of a prescribed temperature step for which the response time will indicate its experimental adequacy; rather, actual signals are those that are obtained by convoluting the true signal with the response of the sensor to the unit impulse function. In this paper, we examine not only the time constant of the sensor but also the response function itself, which can be used to determine the actual time evolution of the measured signal by deconvolution procedures.

## Summary and Conclusions

The time response of a solid-embedded thermocouple was analyzed based on a finite differences heat transfer numerical solution. The analysis is given for a steplike temperature change of the measured domain that results in the most extreme case of time delay for a solid-embedded thermocouple.

It was found that in cases where the thermal diffusivity of the thermocouple is at least one order of magnitude higher than that of the measured domain, (1) the temperature distribution within the cross section of the thermocouple wires can be taken as uniform from heat transfer considerations after one time constant; (2) the transient response is governed by the thermocouple radius and the thermal diffusivity of the measured domain, regardless of the thermal diffusivity of the thermocouple; (3) the approximation of the thermocouple wires as infinite cylinders leads to somewhat longer time constants than those in the case of approximating the thermocouple junction as a sphere; (4) the measured domain can be treated as infinite from heat transfer considerations when its radius is larger than three times the thermocouple radius. In summary, the thermal diffusivity of the thermocouple needs to be at least one order of magnitude higher than that of the measured domain in order to monitor adequately its transient response.

Unlike the transient response of a fluid-immersed thermocouple, and in contrast to common belief, the time response of a solid-embedded thermocouple is far from being similar to that of a first-order process. An approximated response function is suggested based on numerical solution results and a least squares approximation technique. It has been found that the response function is nearly exponentially dependent on the square root of the Fourier number of the measured domain. It follows that, with respect to fluid temperature measurements, a much faster time response is expected at the initiation of the process on one hand, and a much longer time period is required for reaching a steady state on the other hand.


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