by M. Vural and D. Rittel

AN EDUCATIONAL VISUALIZATION TECHNIQUE FOR KOLSKY (SPLIT HOPKINSON) BAR

The present paper introduces a simple educational technique that allows the design of a variety of creative and instructive experiments in the areas of one dimensional wave motion in solids and dynamic behavior of materials. The technique is essentially based on the Kolsky (split Hopkinson) pressure bar setup. However, instead of electronic data acquisition tools, it uses a block of plasticine (modeling clay) as the basic tool to capture, visualize and quantify the displacement (or stress) waves, thus making the physical process of transient wave propagation easily conceivable particularly for the students. When coupled with the elementary theory of stress wave propagation in long bars the technique is shown to be capable of providing useful information about the average flow stress of materials at varying strain rates.

INTRODUCTION

The propagation of elastic waves in solids has long been an area of interest because stress waves provide a powerful tool for studying the mechanical properties of solids and also due to the fact that the understanding of high-strain-rate behavior of materials has become increasingly important from an engineering point of view.[1]
Bertram Hopkinson[2] was among the first to investigate experimentally the propagation of stress pulses in long bars on a laboratory scale. His apparatus, which has become known as the Hopkinson pressure bar, is an application of the simple theory of stress propagation of elastic pulses in a cylindrical bar where the length of the pulse is large when compared to transit time of an elastic wave across the radius of the bar. A significant feature of this work was that the pressure pulses propagating along the bar were estimated by measuring the momentum (with a ballistic pendulum) acquired by momentum traps of different lengths placed in contact with the bar at the far end. Later, an electrical version of the Hopkinson bar devised by Davies[3] used condensers to continuously measure displacements in the pressure bar. Kolsky[4] introduced the novel Kolsky (split Hopkinson) pressure bar technique, in which the specimen is sandwiched and dynamically compressed between two pressure bars. He followed Davies'[3] technique to measure the displacement profiles in both bars and demonstrated, through a one-dimensional elastic wave analysis, how stress and strain within the deforming specimen are related to displacements in the pressure bars.

Since then there have been numerous advances in experimental procedures based on the Hopkinson bar. Novel modifications of the two-bar configuration have been introduced to subject the specimen to dynamic uniaxial stress[5,6] and torsional loading.[7] For a review of these classical techniques and further modifications for specific applications, the reader is referred to ASM Handbook.[8]

Common to all these dynamic test techniques is that the specimen is loaded through the passage of a finite-length stress wave guided by long bars. The propagation of stress pulses along the long bars, their reflection and interaction are classical problems extensively treated in textbooks (e.g., Kolsky,[9] Achenbach,[10] Graff,[11] Meyers[1]). It is well known that the motion of a long bar subjected to a pulse loading from one end does not occur in a uniform manner, i.e., particle velocity is not ubiquitous and induced only in the portions of the bar where the propagating stress pulse is present at that particular time. Since the time scale of this phenomenon is too short for visual observation, an analogy with the worm/snake movement is frequently established in the textbooks to depict the actual physical process and make it easily conceivable by the reader. The usual means of detecting this temporal process is to use strain gages and an expensive high-speed oscilloscope which may not exist in every laboratory. Moreover, a direct visualization of the bar motion is not readily available in the literature, even in the form of stroboscopic recording. The present study introduces a simple and yet powerful technique to capture and visualize the signature of this worm-like movement. It also shows that the technique can easily be calibrated for instructive experiments.

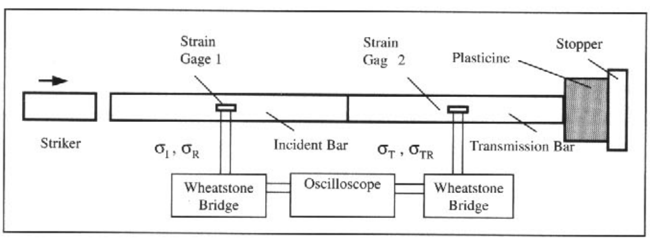

Fig. 1: Schematic of the Kolsky pressure bar setup used in the experiments

EXPERIMENTAL SETUP

The dimensions of the bars in the Kolsky pressure bar used are 305 mm, 1250 mm and 1200 mm in length for the striker, incident and transmission bars respectively, with a common diameter of 12.7 mm. Figure 1 shows the schematic of the experimental setup used. All the bars are made of C350 Maraging steel with Young's modulus (E) and longitudinal wave velocity (C) of 190 GPa and 4877 m/s respectively. High resistance (1000 Ω) strain gages (Micro-measurements, WK-06-250BF-10C) are used with an excitation voltage of 30 volts to measure the surface strains on incident and transmission bars. The raw strain gage signals are recorded using a high-speed 12-bit digital oscilloscope (Nicolet 440). A block of oil-based, non-hardening plasticine (modeling clay, Van Aken International, Rancho Cucamonga, CA) is used just in front of the stopper plate and in contact with the far end of

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the transmission bar to capture the signature of intermittent end displacements.

WAVE PROPAGATION AND DISPLACEMENT SIGNATURE

A gas gun is used to accelerate the striker, which eventually impacts the one end of the incident bar and a stress pulse starts propagating along the incident bar. The length of the stress pulse is twice the transit time along the length of the striker bar and its magnitude depends on the impact velocity, which in turn is a function of the pressure in the gas chamber. This compression, pulse recorded by the strain gage on the incident bar, is transmitted to the second bar through the contact surface and recorded once more by the strain gage on the transmitted bar. During its propagation, the compressive stress pulse induces a particle velocity in the direction of propagation, which is given by

$$V_T(t) = \frac{|\sigma_T(t)|}{\rho C} = \frac{|\sigma_T(t)}{\sqrt{\rho E}} \quad \text{and} \quad \sqrt{E/\rho} \tag{1}$$

where $\sigma_T(t)$, ρ, C and E are the stress profile, density, longitudinal wave velocity and Young's modulus of the bar material, respectively. When the compression pulse, $\sigma_T(t)$, reaches the far end of the transmission bar it reflects back as a tension pulse, $\sigma_{TR}(t)$. Actually the boundary condition at the far end of the transmission bar is not that of a free surface because it is in contact with the plasticine. However, since the acoustic impedance of plasticine is very small in comparison to that of the steel bar, we can assume for all practical purposes to have a free surface, an assumption that is justified by the comparison of the strain gage signal of the incoming pulse with that of the reflected stress pulse. Therefore, the reflected tension pulse induces a particle velocity, $V_{TR}(t) = V_T(t)$, which is in opposite direction to that of propagation, and the resulting particle velocity at the bar-plasticine contact interface doubles in its value, $V_E(t) = 2V_T(t)$, during the reflection. When the stress pulse is completely reflected the bar-plasticine interface stops moving and stands still until the next pulse arrives. When the reflected tension pulse reaches the interface between incident and transmission bars, the interface cannot sustain the tension and the pulse reflects back as a compression pulse. Hence the entire cycle described above repeats itself over and again until the far end of the transmission bar hits the stopper plate.

In each cycle the bar-plasticine interface displaces by an amount Δx which is given by,

$$\Delta x = \int_0^\tau V_E(t)\,dt = \int_0^\tau \{V_T(t) + V_{TR}(t)\}\,dt$$
$$= \int_0^\tau 2V_T(t)\,dt = \int_0^\tau 2\,\frac{|\sigma_T(t)|}{\rho C}\,dt \tag{2}$$

where τ is the duration of the stress pulse generated, given by $2L_S/C$, where L_S is the length of the striker bar. The time period between these cyclic displacements is given by,

$$\Delta t = \frac{2L_T}{C} - \tau \tag{3}$$

where L_T is the length of transmission bar. During this time period the bar-plasticine interface does not move and then displaces once more for a time period of τ, completing one cycle. Therefore, each cycle corresponds to a penetration depth of Δx into the plasticine material and, due to the viscoplastic nature of plasticine response, leaves a permanent signature on the inner surface of the penetration hole in the form of circular rings.

This generic ring pattern is shown in Fig. 2 together with the recorded strain (stress) signal and calculated displacement history. Figure 2a shows the stress history measured by the strain gage on the transmission bar. Figure 2b is the plot of displacement history at the bar-plasticine interface calculated according to Eq. (2) by using the real stress profiles measured, i.e., $V_E(t) = V_T(t) + V_{TR}(t)$. For comparison purposes, the displacement history was also calculated by taking $V_E(t) = 2V_T(t)$ and plotted in Fig. 2b. However, the difference between the two cannot be seen because they overlap perfectly. The incremental displacements denoted by Δx are 2.3 mm. Finally, Fig. 2c shows the inner surface of the penetration hole in the plasticine block recovered after being subjected to the stress and displacement histories shown in Figs. 2a,b. The development of this periodic ring pattern is a combined result of intermittent displacement history and the viscoplastic nature of plasticine. In order to improve the visual quality of the ring pattern, the far end of the transmission bar has been heated for a few seconds, prior to putting it in contact with the plasticine, by using a gasoline pocket lighter so that a thin layer of soot has been deposited on the bar surface. Each ring corresponds to an incremental movement of the bar-plasticine interface. The width of the rings averaged over the first three rings was measured to be $\Delta x_m = 2.2$ mm, with only 4 percent deviation from the value $\Delta x = 2.3$ which was calculated based on the recorded stress profiles. Successive experiments show that the displacement results are reproducible within the same accuracy band. Even though the dissipative effects are insignificant at the early stages of penetration it should be noted that there is a gradual decrease in ring width as the penetration proceeds. Figure 3 further illustrates the captured displacement (stress) waves for different impact velocities. The experimental conditions such as the striker velocity are noted in the figure captions. Note that the predictions for impact velocity (or the amplitude of stress pulse) based on the plasticine signature are in close agreement to those calculated by strain gage measurements.

The significance of this simple visualization technique has two aspects. First, it is easy to implement and gives the student a concrete understanding of wave motion in solids through the visual detection of a physical process that takes place in a small time scale of the order of the microsecond. Second, it allows the design of simple, creative and instructive experiments concerning both the wave propagation and the dynamic behavior of materials, which will be the subject of the following section.

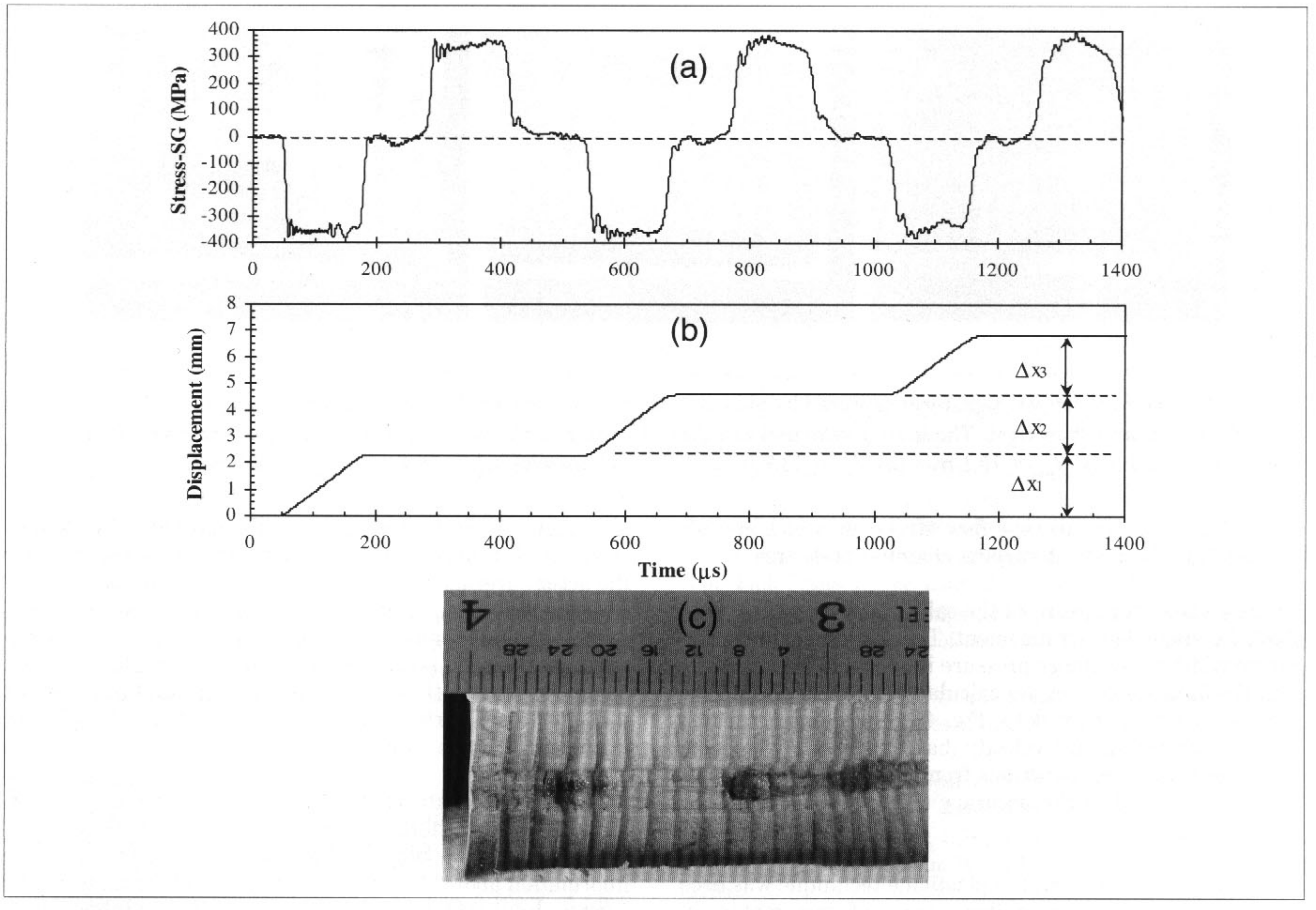

Fig. 2: (a) Plot of stress pulse profile recorded by the strain gage on transmission bar, (b) calculated displacement history at the bar-plasticine interface and (c) cross-sectional view of the recovered plasticine after being subjected to the displacement history shown in (b), penetration into the plasticine is from left to right. Note that each division on the ruler corresponds to 0.79 mm.

CALIBRATION AND MATERIALS TESTING

Based on the plasticine technique, a brief treatment of the Kolsky (split Hopkinson) pressure bar testing of materials is presented in the following. The width of displacement rings is the only *measurable* quantity in this technique. For any stress pulse of known wave length this information can be used, through Eq. (2), to extract the average magnitude of stress pulse and the impact velocity which are given by

$$\sigma_T^{\mathrm{av}} = \frac{\Delta \times E}{5L_S} \tag{4a}$$

$$V_{\mathrm{im}} = \frac{\Delta \times C}{2L_S} \tag{4b}$$

where L_S is the length of striker bar. It is clear that as long as the stress profile is approximately square in its form, which is the case for most pulses generated by the impact of a striker bar of reasonable length, the average stress calculated by Eq. (4) will give the real magnitude of pulse.

Once the technique is calibrated with a single bar (e.g., incident bar only) so as to provide a calibration curve between chamber pressure (P) and the magnitude of stress pulse generated (σ_I), it can be used to perform simple and instructive experiments towards investigating, say, the average flow stress of metals. If the specimen under consideration is placed in between the incident and transmission bars and the striker bar is accelerated with a known chamber pressure, one will have the information about the magnitude of both incident pulse (σ_I) and transmitted pulse (σ_T), the former from calibration curve and the latter from the first displacement signature in plasticine. By assuming that the stress equilibrium within the specimen is reached, average strain rate and flow stress experienced by the specimen can be found as in the following

$$\dot{\varepsilon} = \frac{2(\sigma_I - \sigma_T)}{\rho C l_0} = \frac{E}{2\rho C l_0 L_S}(\Delta x' - \Delta x) \tag{5a}$$

$$\sigma_{\mathrm{flow}} = \frac{\Delta E A_b}{4 L_S A_0} \tag{5b}$$

where l_0 is the specimen length, $\Delta x'$ is the displacement signature from the calibration test obtained with the same chamber pressure as used in the current test, Δx is the displacement signature from the current test, A_b and A_0 are the cross-sectional areas of the bar and specimen respectively. With this simple methodology one can easily construct av-

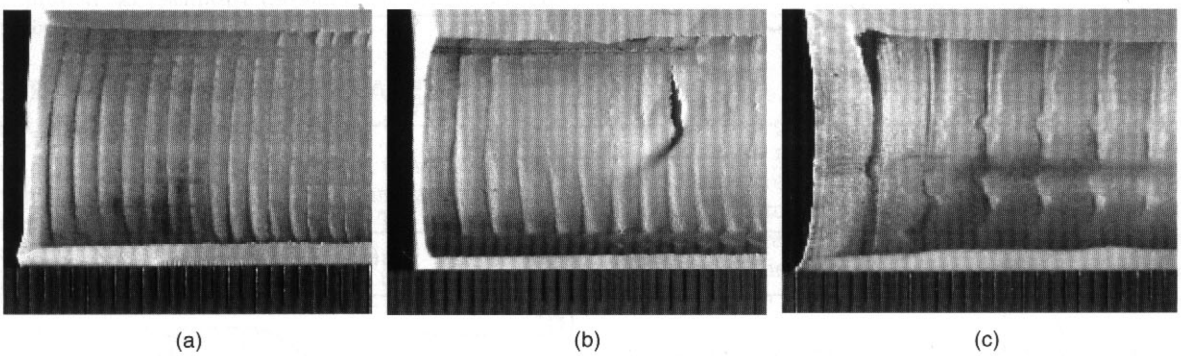

Fig. 3: Displacement (stress) signatures captured by plasticine at varying impact velocities for a striker length of 305 mm. Penetration is from left to right. The impact velocities calculated by using strain gage signals (V_{sg}) and plasticine signatures (V_{ps}) are: (a) V_{sg} = 9.9 m/s, V_{ps} = 10.3 m/s (b) V_{sg} = 13.9 m/s, V_{ps} = 14.1 m/s (c) V_{sg} = 25.1 m/s, V_{ps} = 25.3 m/s.

erage strain-rate vs. average flow stress plots of materials by repeating the tests at varying chamber pressures.

Figure 4 shows the results of the calibration study described above for single bar arrangement. The plot of plasticine signature width vs. chamber pressure is shown in Fig. 4a along with the displacement cycles calculated by using strain gage signals. Based on these data, Fig. 4b shows the correlation between the real impact velocity (based on strain gage measurements) and the predictions from plasticine signature. It is worth noting that the accuracy of predictions is always within 5 percent.

Following the calibration, the plasticine technique was used to determine the average flow stress of two materials; C11000 copper and Al 6061-T6 alloy. The cylindrical specimens tested have the dimensions of ϕ (diameter) = l_0 (length) = 9.52 mm. The dotted lines in Fig. 5 show the average flow stress of these materials as calculated from plasticine signatures. The stress-strain plots of materials are also shown for comparison. Even though the average flow stress predictions of the plasticine technique lie slightly over the actual average values, nevertheless they are able to capture the difference in flow stress, which is as low as 10 percent. It should be noted that when making this type of comparisons the total strain experienced by the materials must be approximately the same. This criterion can be easily met if the tests are performed with the same striker length and at the same strain rate.

For one kind of material, the stress-strain curve may be constructed using different striker bars at the same velocity. These tests, while keeping the strain rate constant, provide information about the average flow stress of the material for varying total strains so that the hardening modulus can be obtained. On the other hand, the plasticine technique is not suitable to determine the strain-rate sensitivity of materials. When testing a material at increasing strain rates the total strain also increases, and if the material tested is strain hardening this situation results in an increase in the aver-

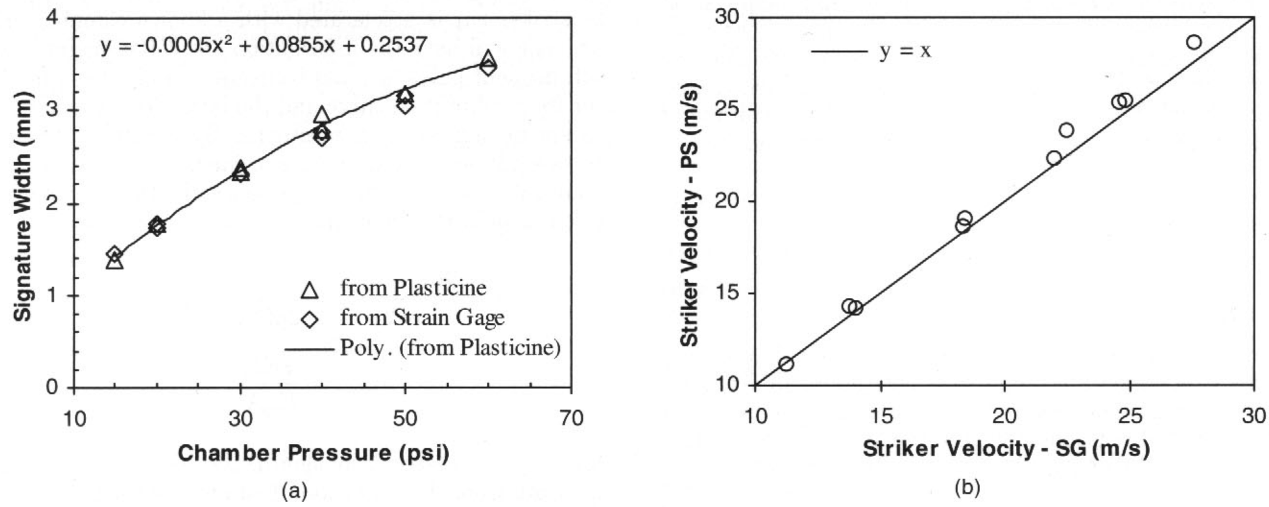

Fig. 4: (a) Calibration curve gives the relation between chamber pressure and plasticine signature width. Displacement cycles calculated by strain gage data are also plotted for comparison; (b) Correlation between real impact velocity (SG) and the predictions from plasticine signature (PS).

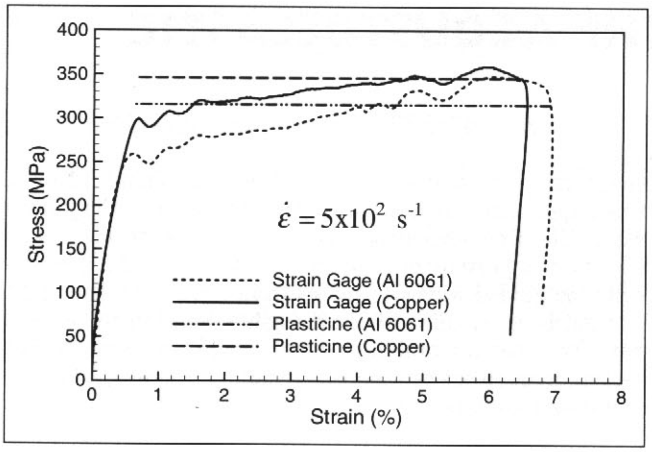

Fig. 5: The stress-strain curves for copper and aluminum alloy at a strain rate of $5 \times 10^2\ s^{-1}$. Horizontal lines represent the average flow stress as calculated from the plasticine signature.

age flow stress even if the material does not have any strain-rate sensitivity. However, this problem may be handled by using strikers of different lengths at increasing strain rates and through meticulous calibration for varying striker lengths.

SUMMARY

A simple experimental technique is introduced that allows the design of a variety of creative and instructive experiments in the areas of one dimensional wave motion in solids and dynamic response of materials. It uses a block of plasticine (modeling clay) as the basic tool to capture, visualize and quantify the displacement (or stress) waves, thus making the physical process of transient wave propagation easily conceivable particularly for the students. When coupled with the elementary theory of stress wave propagation in long bars the technique is shown to be capable of providing useful information about the average constitutive response of materials at high strain rates. However, the application of technique towards investigating the strain-rate sensitivity of materials requires additional and meticulous calibration for varying striker lengths.

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